Eigenvalues Eigenvectors

~~Eigen Value~~ (~~eige~~ e.values), ~~Eigen Vectors~~ and One of Their Applications

Reference. 北大《高等代数》, Sec 7.4. "特征值与特征向量"。

$B = X^{-1}AX \Leftrightarrow$

A, B are the same transformation (linear) under dff bases.

$\downarrow$

Motivation: "Can we find a "good" basis to express $\mathcal{L}$ in a diagonal matrix?"

$\downarrow$

Diagonal Λ

$\downarrow$

Benefit: e.values are the main diagonal entries (the same for block-diagonal matrices

Linear transformation $\mathcal{L}$

$\downarrow$

A

① $\downarrow$ $\det(\lambda I - A) = 0$

$\lambda \longrightarrow V_\lambda = \{x \in \mathbb{R}^n \mid Ax = \lambda x\}$

②

③

$$\boxed{A \sim \Lambda \Leftrightarrow \sum_i \dim(V_{\lambda_i}) = n}$$

Note ①. Although we get λ by solving $\det(\lambda I - A) = 0$, the values of λ depends solely on $\mathcal{L}$ (not the choices of basis and thus A).

Note ②. The eigenvectors correspond to λ is not a single one; they constitute a ~~ve~~ subspace V_λ of $\mathbb{R}^n$.

Note ③. Two important facts to prove:

(a) If $v_1 \in V_{\lambda_1}$, $v_2 \in V_{\lambda_2}$, $\lambda_1 \neq \lambda_2 \Rightarrow v_1$ & v_2 are linearly independent.

(b). $\sum_i$ ~~V_{λ_i}~~ $\dim(V_{\lambda_i}) \sim (A \sim \Lambda)$.

How to relate to